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ON A CONJECTURE OF C. A. MICCHELLI CONCERNING CUBIC SPLINE INTERPOLATION AT A BIINFINITE KNOT SEQUENCE

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ABSTRACT

It is shown that if the knot sequence $\underline{t}:= (t_i)_{-\infty}^{\infty}$ satisfies

(i) For some $m \in \left[1, \frac{3+\sqrt{5}}{2}\right)$,

$$m^{-1} < \underline{\lim}_r \inf_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} < \overline{\lim}_r \sup_i \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} < m$$

and

(ii) $$m_{\underline{t}} := \sup_{|i-j|<1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} < \infty \quad ,$$

then for any given bounded sequence $y \in m(\mathbb{Z})$ there exists exactly one cubic spline s with knots t_i such that

$$s(t_i) = y_i \quad , \quad \text{for all } i \in \mathbb{Z} \quad .$$

AMS (MOS) Subject Classification: 41A15

Key Words: cubic spline interpolation, exponential decay, Lagrange's multiplier

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SIGNIFICANCE AND EXPLANATION

Cubic spline interpolation provides a good and handy method to approximate a given function or to fit a given set of points. However, such an interpolation process does not always converge. It is known that the local mesh ratio (that of the lengths of two consecutive intervals) is less than $\frac{3+\sqrt{5}}{2}$, the interpolation process works for any given bounded data.

This paper continues such investigation. It is shown that the above restriction on the knots may be relaxed. Thus, for a wider class of knot sequences, the cubic spline interpolation can be still applied. Hopefully, this would make such interpolation process more feasible in practice.

3 + sq. root / 2

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

ON A CONJECTURE OF C. A. MICCHELLI CONCERNING CUBIC SPLINE
INTERPOLATION AT A BIINFINITE KNOT SEQUENCE

Rong-qing Jia

1. Introduction. Let $\underline{t} := (t_i)_{-\infty}^{\infty}$ be a biinfinite, strictly increasing sequence, set

$$t_{\pm\infty} := \lim_{i \to \pm\infty} t_i \quad ,$$

let k be an integer, $k \geq 2$, and denote by $\$_{k,\underline{t}}$ the collection of spline functions of degreee $< k$ with knot sequence $\underline{t}$. Explicitly, $\$_{k,\underline{t}}$ consists of exactly those $k-2$ times continuously differentiable functions on

$$I := (t_{-\infty}, t_{\infty})$$

which, on each interval (t_i, t_{i+1}), coincide with some polynomial of degree $< k$. Let

$$m\$_{k,\underline{t}} := \$_{k,\underline{t}} \cap m(I) \quad ,$$

i.e., the normed linear space of splines for which

$$\|s\|_{\infty} := \sup_{t \in I} |s(t)|$$

is finite. We are interested in the

Bounded Interpolation Problem (B.I.P.). To construct, for given $y \in m(\mathbb{Z})$, some $s \in m\$_{k,\underline{t}}$ for which

$$s|_{\underline{t}} = y \quad . \tag{1}$$

We will say that the B.I.P. is correct (for the given knot sequence $\underline{t}$) if it has exactly one solution for each $y \in m(\mathbb{Z})$.

In case $k = 4$ (cubic spline interpolation), de Boor [2] showed that if the local mesh ratio

$$m_{\underline{t}} := \sup_{|i-j| \leq 1} \frac{\Delta t_i}{\Delta t_j} \quad (\text{with } \Delta t_i := t_{i+1} - t_i)$$

is less than $\frac{3+\sqrt{5}}{2}$, then the B.I.P. is correct. A similar result was also obtained independently by Zmatrakov [7]. The basic idea of [2] was the exponential decay law, which

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could be traced back to [1]. This idea was developed in de Boor [3] and Micchelli [6]. For cubic spline interpolation, Micchelli further raised the following conjecture (see [6]; p. 236).

Conjecture. If

$$(2) \qquad m^{-1} \le \underline{\lim_{r}} \inf_{i} (\Delta t_{i+r}/\Delta t_{i-1})^{\frac{1}{r}} \le \overline{\lim_{r}} \sup_{i} (\Delta t_{i+r}/\Delta t_{i-1})^{\frac{1}{r}} \le m$$

for some $m \in [1, \frac{3+\sqrt{5}}{2})$, then the B.I.P. is correct for $k = 4$.

In hindsight, it is easy to see that this conjecture is faulty. This can be clarified by the following

Example. Write $h_i := t_{i+1} - t_i$. Let

$$t_0 = 0$$

$$h_{2n} = (\tfrac{1}{2})^n, \; h_{2n+1} = (\tfrac{1}{2})^{2n+1} \quad \text{for integer} \quad n \ge 0 \quad ,$$

$$h_{-1-i} = h_i \quad \text{for integer} \quad i \ge 0 \quad .$$

Then

$$\frac{1}{2} \le \underline{\lim_{r}} \inf_{i} \left(\frac{h_{i+r}}{h_{i-1}}\right)^{\frac{1}{r}} \le \overline{\lim_{r}} \sup_{i} \left(\frac{h_{i+r}}{h_{i-1}}\right)^{\frac{1}{r}} \le 2 \quad ,$$

but

$$m_{\underline{\underline{t}}} = \sup_{|i-j| \le 1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} = \infty \quad .$$

Let m^+ and m^- have the same meaning as in [4]. Since the knot sequence $\underline{\underline{t}}$ is symmetric with respect to the origin, we must have $m^+ = m^-$. If this B.I.P. is correct, then [4] tells us that $m_{\underline{\underline{t}}} < \infty$, which is a contradiction.

The above example suggests to us that the condition $m_{\underline{\underline{t}}} < \infty$ should be added to the assumption. Thus we will prove the following

Theorem 1. If a knot sequence $\underline{\underline{t}}$ satisfies (2) with $1 \le m < (3+\sqrt{5})/2$ and

$$(3) \qquad m_{\underline{\underline{t}}} < \infty \quad ,$$

then the B.I.P. is correct for $k = 4$.

Remark. This theorem covers de Boor's results for cubic splines with bounded global mesh ratio or with local mesh ratio $< \frac{3+\sqrt{5}}{2}$ (see [2] and [3]).

2. <u>The basic formulae for cubic spline interpolation</u>. For a given knot sequence $\underset{\approx}{t}$, let

(4) $$h_i := t_i - t_{i-1},\ \lambda_i = h_{i+1}/(h_i + h_{i+1}),\ \mu_i = h_i/(h_i + h_{i+1}),\ i \in \mathbb{Z} .$$

If $s \in m\$_{4,\underset{\approx}{t}}$ satisfies (1) for some $y \in m(\mathbb{Z})$, then

(5) $$\lambda_i s'(t_{i-1}) + 2s'(t_i) + \mu_i s'(t_{i+1}) = 3\lambda_i \frac{y_i - y_{i-1}}{h_i} + 3\mu_i \frac{y_{i+1} - y_i}{h_{i+1}},\ i \in \mathbb{Z} .$$

Moreover, it is easy to check that

$$s(x) = \frac{(x-t_{i-1})(t_i-x)^2}{h_i^2} s'(t_{i-1}) - \frac{(x-t_{i-1})^2(t_i-x)}{h_i^2} s'(t_i) + \frac{y_{i-1} + y_i}{2} +$$

(6) $$+ \frac{1}{h_i^2}\left(x - \frac{t_{i-1} + t_i}{2}\right)[(t_i-x)^2 + 4(t_i-x)(x-t_{i-1}) + (x-t_{i-1})^2] \frac{y_i - y_{i-1}}{h_i}$$

$$\text{for } x \in [t_{i-1}, t_i] .$$

Let

(7) $$\forall i \in \mathbb{Z},\ A(i,j) := \begin{cases} 2 & \text{for } j = i , \\ \lambda_i & \text{for } j = i-1 , \\ \mu_i & \text{for } j = i+1 , \\ 0 & \text{for } j \in \mathbb{Z}\setminus\{i-1,i,i+1\} . \end{cases}$$

Then A is a tri-diagonal $\mathbb{Z} \times \mathbb{Z}$-matrix. For any $\beta \in m(\mathbb{Z})$ we have

$$\forall i \in \mathbb{Z},\ |\lambda_i\beta_{i-1} + 2\beta_i + \mu_i\beta_{i+1}| \le \lambda_i \|\beta\|_\infty + 2\|\beta\|_\infty + \mu_i \|\beta\|_\infty = 3\|\beta\|_\infty ,$$

showing $\|A\| \le 3$. Here, we view A as a mapping from $m(\mathbb{Z})$ to $m(\mathbb{Z})$. Furthermore,

$$\forall i \in \mathbb{Z},\ |\lambda_i\beta_{i-1} + 2\beta_i + \mu_i\beta_{i+1}| \ge 2|\beta_i| - \lambda_i\|\beta\| - \mu_i\|\beta\| = 2|\beta_i| - \|\beta\|_\infty .$$

Hence $\|A\beta\|_\infty \ge \sup_{i\in\mathbb{Z}}\{2|\beta_i| - \|\beta\|_\infty\} = \|\beta\|_\infty$. This shows that A^{-1} exists and $\|A^{-1}\| \le 1$.

3. <u>The exponential decay</u>. The following lemma plays an essential role in this paper.

Lemma 1. For any knot sequence $\underset{\approx}{t}$,

(8) $$|A^{-1}(j,i)| \le 2^{-|j-i|},\ \forall i, j \in \mathbb{Z} ,$$

and

(9) $$A^{-1}(j,i)A^{-1}(j+1,i) \le 0,\ \forall i, j \in \mathbb{Z} .$$

Moreover, if $\underline{\underline{t}}$ satisfies the following condition: for some integer $r > 0$

(10) $$h_i/h_j < \frac{3}{4} m_0^{|i-j|} \quad \text{whenever} \quad |i-j| > r \ ,$$

then

(11) $$|A^{-1}(j,i)| < \left(1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}}\right)^{-|i-j|} \quad \text{whenever} \quad |i-j| > r \ .$$

Proof. For simplicity we fix i and write $b_j := A^{-1}(j,i)$. Since $\|A^{-1}\| < 3$, $b(\cdot) \in m(\mathbb{Z})$. By $AA^{-1} = 1$ we have

(12) $$\lambda_j b_{j-1} + 2b_j + \mu_j b_{j+1} = \delta_{ij} = \begin{cases} 1 & \text{for } j = i \ , \\ 0 & \text{for } j \neq i \ . \end{cases}$$

We claim that

(13) $$|b_j| < |b_{j+1}| \quad \text{for all} \quad j < i \ .$$

If not, then there exists $j_0 < i$ such that $|b_{j_0}| > |b_{j_0+1}|$. From (12) we have

$$|b_{j_0-1}| = |-(2b_{j_0} + \mu_{j_0} b_{j_0+1})/\lambda_{j_0}| > (2|b_{j_0}| - \mu_{j_0}|b_{j_0+1}|)/\lambda_{j_0}$$

$$> (2 - \mu_{j_0})|b_{j_0}|/\lambda_{j_0} = |b_{j_0}|\cdot(1 + \lambda_{j_0})/\lambda_{j_0} > 2|b_{j_0}| \ .$$

Then by induction on j, we can easily show that $|b_{j-1}| > 2|b_j|$ for all $j < j_0$. Hence $|b_j| > 2^{j_0-j}|b_{j_0}|$ for $j < j_0$. This contradicts the fact $b \in m(\mathbb{Z})$. Similar to (13), the following also holds:

(13') $$|b_j| < |b_{j-1}| \quad \text{for all} \quad j > i \ .$$

Now (12) and (13) yield that

(14) $$\begin{aligned} |b_{j+1}| &= |2b_j + \lambda_j b_{j-1}|/\mu_j > (2|b_j| - \lambda_j|b_{j-1}|)/\mu_j \\ &> (2-\lambda_j)|b_j|/\mu_j = |b_j|\cdot(1 + \mu_j)/\mu_j > 2|b_j| \quad \text{for} \quad j < i \ . \end{aligned}$$

Similarly

(14') $$|b_{j-1}| > 2|b_j| \quad \text{for} \quad j > i \ .$$

In particular, $|b_{i-1}| < |b_i|$ and $|b_{i+1}| < |b_i|$. In connection with (12) we obtain

(15) $$\begin{aligned} 1 &= \lambda_i b_{i-1} + 2b_i + \mu_i b_{i+1} = |\lambda_i b_{i-1} + 2b_i + \mu_i b_{i+1}| > \\ &> 2|b_i| - \lambda_i|b_{i-1}| - \mu_i|b_{i+1}| > (2 - \lambda_i - \mu_i)|b_i| = |b_i| \ . \end{aligned}$$

This proves (8) for $j = i$. For $j \neq i$, (8) comes from (14), (14') and (15). For the rest of the proof we may assume $j < i$ without any loss. To prove (9) we argue indirectly. If $b_{j_0}b_{j_0+1} \geq 0$ for some $j_0 < i$, then

$$|b_{j_0-1}| \geq |\lambda_{j_0} b_{j_0-1}| = |2b_{j_0} + \mu_{j_0} b_{j_0+1}| \geq 2|b_{j_0}| \quad .$$

Comparing the above inequality with (13), we must have $b_j = 0$ for all $j \leq j_0$. It would cause all $b_j = 0$, which is absurd. Now we can write down

$$(16) \qquad -b_{j+1}/b_j =: 2 + 2q'_j \quad \text{for} \quad j < i$$

with $q'_j \geq 0$. Let $q_j := h_{j+1}/h_j$. We deduce from (12) that, for $j < i-1$,

$$-\frac{b_{j+2}}{b_{j+1}} = \frac{2b_{j+1} + \lambda_{j+1}b_j}{\mu_{j+1}b_{j+1}} = \frac{2}{\mu_{j+1}} + \frac{\lambda_{j+1}}{\mu_{j+1}}\cdot\frac{b_j}{b_{j+1}} = 2 + 2q_{j+1} + q_{j+1}\cdot\frac{-1}{2+2q'_j}$$

$$= 2 + 2q_{j+1}\left(1 - \frac{1}{2(2+2q'_j)}\right) = 2 + 2q_{j+1}\cdot\frac{4q'_j + 3}{4q'_j + 4} \quad .$$

This shows that

$$(17) \qquad q'_{j+1} = q_{j+1}\cdot\frac{4q'_j + 3}{4q'_j + 4} \quad \text{for} \quad j < i-1 \quad .$$

Let

$$(18) \qquad p_j := \frac{4q'_j + 4}{4q'_j + 3}\, q'_j \quad .$$

It is easy to verify that

$$(19) \qquad 2 + 2q'_j = 1 + p_j + \sqrt{1 + p_j + p_j^2} \quad .$$

Now (16) and (19) give us

$$(20) \qquad |b_i/b_j| = \prod_{k=j}^{i-1} |b_{k+1}/b_k| = \prod_{k=j}^{i-1} (2 + 2q'_k) = \prod_{k=j}^{i-1} (1 + p_k + \sqrt{1 + p_k + p_k^2})$$

It follows from (17) and (18) that

$$(21) \qquad \begin{aligned} \prod_{k=j}^{i-1} p_k &= \prod_{k=j}^{i-1} \left(\frac{4q'_k + 4}{4q'_k + 3}\, q'_k\right) = \prod_{k=j}^{i-1} \left(\frac{4q'_k + 4}{4q'_k + 3}\cdot\frac{4q'_{k-1} + 3}{4q'_{k-1} + 4}\, q_k\right) \\ &= \frac{4q'_{i-1} + 4}{4q'_{i-1} + 3}\cdot\frac{4q'_{j-1} + 3}{4q'_{j-1} + 4}\cdot\prod_{k=j}^{i-1} q_k \geq \frac{3}{4}\prod_{k=j}^{i-1} q_k = \frac{3}{4}\cdot\frac{h_i}{h_j} \quad . \end{aligned}$$

If $\underline{t}$ satisfies (8) and $|i-j| > r$, then

$$\prod_{k=j}^{i-1} p_k > \frac{3}{4} \cdot \frac{h_i}{h_j} > \frac{3}{4} \cdot \frac{4}{3} m_0^{-|i-j|} = m_0^{-|i-j|} \quad . \tag{22}$$

Therefore lemma 1 will be proved, once the following lemma is established:

Lemma 2. Suppose $p_1,\ldots,p_n$ and p are nonnegative real numbers with $p^n = p_1 p_2 \cdots p_n$. Then

$$\prod_{i=1}^{n} (1 + p_i + \sqrt{1 + p_i + p_i^2}) > (1 + p + \sqrt{1 + p + p^2})^n \quad .$$

Proof. Let

$$F(p_1,\cdots,p_{n-1},p_n) := \prod_{i=1}^{n} (1 + p_i + \sqrt{1 + p_i + p_i^2}) \quad .$$

We want to determine the minimnm of the function F under the constraint $\prod_{i=1}^{n} p_i = c$, where c is a constant, $c = p^n$. If some $p_i > (2+2p)^n$, then

$$F(p_1,\cdots p_n) > (2 + 2p)^n > \prod_{i=1}^{n} (1 + p + \sqrt{1 + p + p^2})^n > \inf_{\Pi p_i = c} \{F(p_1,\cdots,p_n)\} \quad .$$

Hence

$$\inf_{\Pi p_i = c} \{F(p_1,\cdots,p_n)\} = \inf_{\substack{\Pi p_i = c \\ \forall i,\, p_i < (2+2p)^n}} \{F(p_1,\cdots p_n)\} \quad .$$

Thus there exists a point $(p_1^0,\ldots,p_n^0)$ with $\prod_{i=1}^{n} p_i^0 = c$ such that $F(p_1^0,\ldots,p_n^0) = \inf_{\Pi p_i = c} \{F(p_1,\ldots,p_n)\}$. To find $(p_1^0,\ldots,p_n^0)$ we shall use the method of Lagrange multipliers and set

$$\Phi(p_1,\cdots p_n) := F(p_1,\cdots p_n) - \lambda p_1 \cdots p_n \quad .$$

Then $\left.\frac{\partial \Phi}{\partial p_i}\right|_{(p_1^0,\cdots,p_n^0)} = 0,\ i = 1,\ldots,n;$ that is

$$\prod_{j \neq i} (1 + p_j^0 + \sqrt{1 + p_j^0 + (p_j^0)^2}) \cdot \left(1 + \frac{2p_i^0 + 1}{2\sqrt{1 + p_i^0 + (p_i^0)^2}}\right) - \lambda \prod_{j \neq i} p_j^0 = 0 \quad .$$

It follows that

$$\frac{p_i^0\left(1 + \dfrac{2p_i^0 + 1}{2\sqrt{1 + p_i^0 + (p_i^0)^2}}\right)}{1 + p_i^0 + \sqrt{1 + p_i^0 + (p_i^0)^2}} = \lambda \cdot \frac{\prod_{j=1}^{n} p_j^0}{\prod_{j=1}^{n} (1 + p_j^0 + \sqrt{1 + p_j^0 + (p_j^0)^2})} \quad .$$

Therefore

$$(23) \qquad f(p_i^0) = f(p_k^0) \quad \text{for all} \quad i, k \in \{1,\dots,n\} \quad ,$$

where

$$f(x) := \frac{x\left(1 + \dfrac{2x+1}{2\sqrt{1+x+x^2}}\right)}{1 + x + \sqrt{1+x+x^2}} = \frac{1}{2} + \frac{x-1}{2\sqrt{1+x+x^2}} \quad .$$

An easy calculation yields

$$(24) \qquad f'(x) = \frac{3(x+1)}{4(1+x+x^2)^{3/2}} > 0 \quad \text{for} \quad x \geq 0 \quad .$$

This shows that f is strictly increasing on $[0,\infty)$. Thus (23) and (24) give

$$p_1^0 = \dots = p_n^0 = p \quad .$$

This ends the proof of lemma 2. Also the proof of lemma 1 is complete.

4. The proof of theorem 1. By the hypothesis (2) there exist a positive integer r and a real number m_0 with $m < m_0 < \frac{3+\sqrt{5}}{2}$ such that

$$h_i/h_j \leq \frac{3}{4} m_0^{|i-j|} \quad \text{whenever} \quad |i-j| \geq r \quad .$$

Then by lemma 1,

$$(25) \qquad \forall i, j \in \mathbb{Z}, \ |A^{-1}(j,i)| \leq \begin{cases} 2^{-|i-j|} & \text{if} \quad |i-j| < r \quad . \\ (1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}})^{-|i-j|}, & \text{if} \quad |i-j| \geq r \quad . \end{cases}$$

Let

$$M := (m_{\underline{t}})^r < \infty$$

and

$$c_i = 3\lambda_i \cdot \frac{y_i - y_{i-1}}{h_i} + 3\mu_i \cdot \frac{y_{i+1} - y_i}{h_{i+1}}, \ i \in \mathbb{Z} \quad .$$

Then it follows that

$$(26) \qquad s'(t_j) = \sum_{i \in \mathbb{Z}} A^{-1}(j,i)c_i = \sum_{|i-j|<r} A^{-1}(j,i)c_i + \sum_{|i-j|\geq r} A^{-1}(j,i)c_i \quad .$$

By the hypotheses of theorem 1 we have the following estimates for c_i:

$$
(27)\begin{cases}
|c_i| \le 6 \cdot \|y\|_\infty \cdot (1+M) \cdot \dfrac{1}{h_i} \le 6M(1+M) \cdot \|y\|_\infty \cdot \dfrac{1}{h_j} & \text{if} \quad |j-i| < r \quad ; \\[2ex]
|c_i| \le 6 \cdot \|y\|_\infty \cdot (1+M) \cdot \dfrac{1}{h_i} \le 6(1+M) \cdot \|y\|_\infty \cdot \dfrac{1}{h_j} \cdot m_0^{|i-j|}, & \text{if} \quad |j-i| \ge r \quad .
\end{cases}
$$

Write

$$
\theta := m_0\left(1 + m_0^{-1} + \sqrt{1 + m_0^{-1} + m_0^{-2}}\right)^{-1} \quad .
$$

Then $\theta < 1$ as long as $m_0 < \frac{3+\sqrt{5}}{2}$. Applying (25) and (27) to (26), we obtain

$$
(28)\quad \begin{aligned}
|s'(t_j)| &\le 6M(1+M) \cdot \|y\|_\infty \cdot \frac{1}{h_j} \cdot \sum_{|j-i|<r} 2^{-|i-j|} + 6(1+M)\|y\|_\infty \cdot \frac{1}{h_j} \sum_{|i-j|\ge r} \theta^{|i-j|} \\
&\le \text{const} \cdot \|y\|_\infty \cdot \frac{1}{h_j} \quad .
\end{aligned}
$$

Furthermore (6) tells us

$$
\max_{t_{j-1} \le x \le t_j} |s(x)| \le \text{const}(h_{j-1}|s'(t_{j-1})| + h_j \cdot |s'(t_j)| + \|y\|_\infty) \quad .
$$

which in connection with (28) yields the desired result

$$
\|s\|_\infty \le \text{const} \cdot \|y\|_\infty \quad .
$$

Our proof is complete.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

cubic spline interpolation, exponential decay, Lagrange's multiplier

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It is shown that if the knot sequence $\underline{\underline{t}} := (t_i)_{-\infty}^{\infty}$ satisfies

(i) For some $m \in \left[1, \frac{3+\sqrt{5}}{2}\right)$,

$$m^{-1} \le \varliminf_{r} \inf_{i} \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} \le \varlimsup_{r} \sup_{i} \frac{t_{i+r+1} - t_{i+r}}{t_{i+1} - t_i} \le m$$

and

(continued)

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ABSTRACT (continued)

(ii) $$m_{\underline{\underline{t}}} := \sup_{|i-j| \leq 1} \frac{t_{i+1} - t_i}{t_{j+1} - t_j} < \infty \ ,$$

then for any given bounded sequence $y \in m(\mathbb{Z})$ there exists exactly one cubic spline s with knots t_i such that

$$s(t_i) = y_i \ , \text{ for all } i \in \mathbb{Z} \ .$$

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